Astrofísica General

24 Mayo 2022

Potencia, P, ó Luminosidad de un Objeto

$$dL = B\,dA\,d\Omega\,d\nu \quad \text{-----} (1)$$

$$B = \frac{dL}{dA\,d\Omega\,d\nu} \quad \text{------} (2)$$

$$[B] = \frac{\text{erg/s}}{\text{cm}^2\ \text{steroradianes}\ \text{Hz}} \quad \text{----} (3)$$

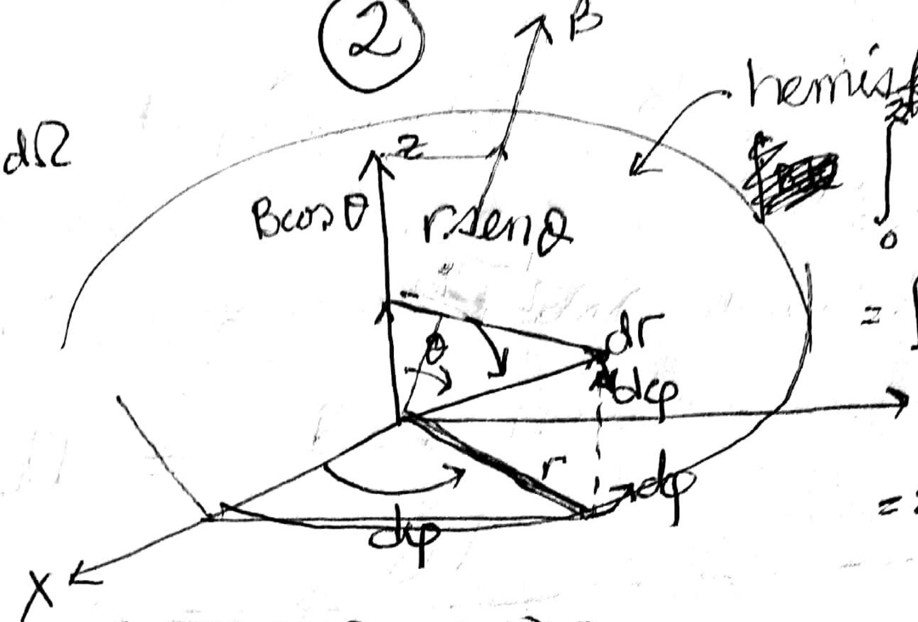
$$\int dL = \iiint_{\nu A \Omega} B\,d\nu\,dA\,d\Omega \quad \text{-----} (4)$$

$$L = \int_A dA \int_\nu B\,d\nu \int_\Omega d\Omega \quad \text{-----} (5)$$

$$L = \iint dA \int_\nu B\,d\nu \iint_{d\theta\, d\varphi} \text{sen}\,\theta\, d\theta\, d\varphi \quad (6)$$

2

$S_\nu = \int B\cos\theta \, d\Omega$



$\int_0^{2\pi}\int_0^{\pi/2} B\cos\theta \, d\Omega$

$= \int_0^{2\pi} B \int_0^{\pi/2} \cos\theta \, sen\theta \, d\theta \, d\varphi$

$= 2\pi B(\nu,T)\int_0^{\pi/2} \frac{1}{2} sen(2\theta)\, d\theta$

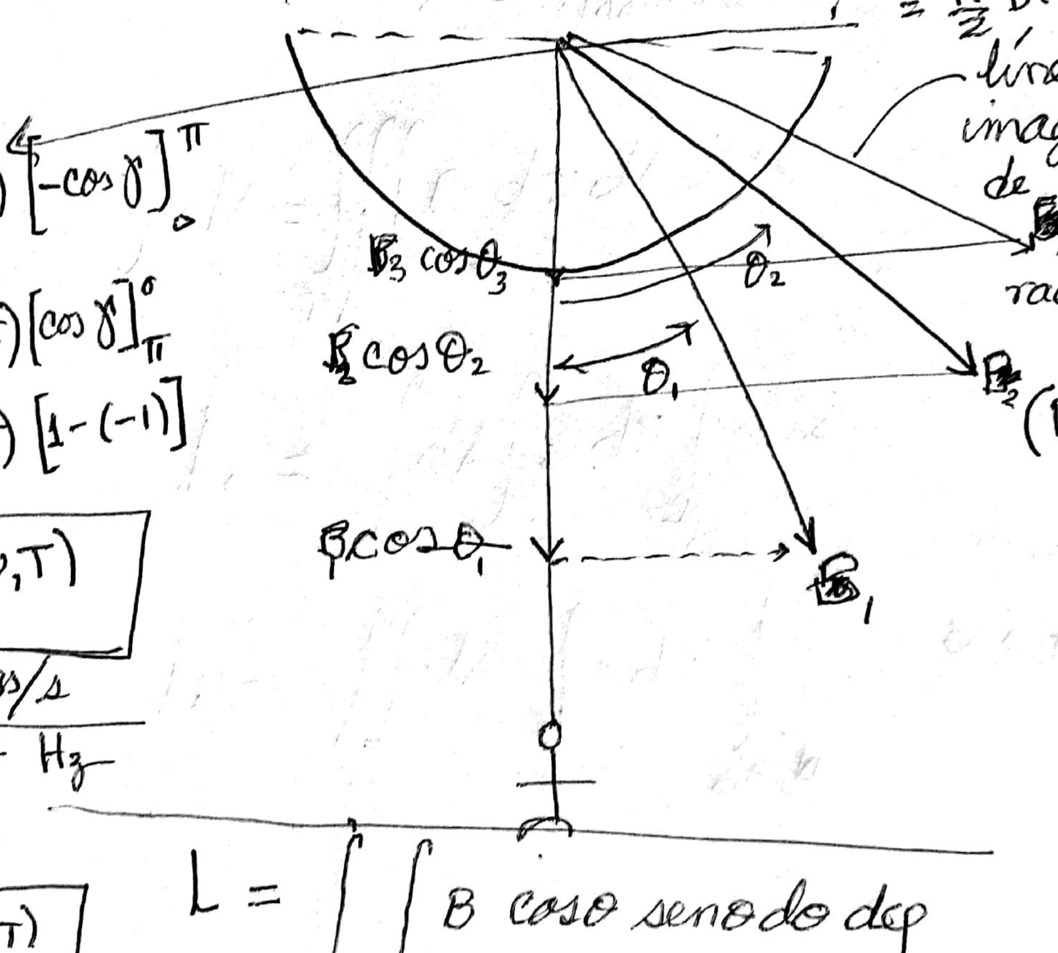
pero $sen\, 2\theta = 2\, sen\theta \cos\theta$

$sen\theta\cos\theta = \frac{1}{2} sen(2\theta)$

$= 2\pi B(\nu,T)\frac{1}{2}\frac{1}{2}\int_0^{\pi/2} sen(2\theta)\, d(2\theta)$

sea $\gamma = 2\theta \therefore \gamma\Big|_0^{\pi}$

$= \frac{\pi}{2} B(\nu,T)\int_0^{\pi} sen\gamma \, d\gamma$



$\frac{\pi}{2} B(\nu,T)\left[-\cos\gamma\right]_0^{\pi}$

$= \frac{\pi}{2} B(\nu,T)\left[\cos\gamma\right]_{\pi}^{0}$

$= \frac{\pi}{2} B(\nu,T)\left[1-(-1)\right]$

$\boxed{S_\nu = \pi B(\nu,T)}$

$[S_\nu] = \frac{ergs/s}{cm^2 \; Hz}$

$L = \int_A \int_\nu B\cos\theta \, sen\theta \, d\theta \, d\varphi$

$\boxed{F_\nu = \pi B(\nu,T)}$

Flujo $F = \int \mathcal{F}_\nu \, d\nu$

$$= \pi \int_0^\infty B(\nu,T)\, d\nu$$ } Emite en todas las frecuencias

$$= \pi \left[\frac{2\pi^4 k^4}{15 c^2 h^3} \right] T^4$$

$$F = \left(\frac{2\pi^5 k^4}{15 c^2 h^3} \right) T^4$$

$$F = \sigma T^4$$

σ: constante de Stephan-Boltzmann

Flujo ó Brillantez de un cuerpo que emite radiación según la Ley de Planck a todas las frecuencias es solamente proporcional a la Temperatura elevada a la cuarta potencia

$$\text{Flujo} = \nu \mathcal{F}_\nu$$
$$= \Delta\nu \, \mathcal{F}_\nu$$
} Flujo ó Brillo a una frecuencia ν ó un intervalo de frecuencias $\Delta\nu$.

4

Función Riemann-Zeta

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$$

Para $z = 4 \rightarrow \zeta(z) = \pi^4/90$

$$\zeta(z) = \frac{1}{\Gamma(z)} \int_0^{\infty} \frac{x^{z-1}}{e^x - 1}\, dx$$

donde $\Gamma(z) = (z-1)!$

$\mathcal{F} \equiv$ total power leaving $1\ cm^2$ of the surface

$\mathcal{F}\left(\frac{ergs/s}{cm^2}\right)$

$$\mathcal{F} = \int_{\nu=0}^{\infty} \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} B(\nu,T) \cos\theta \operatorname{sen}\theta \, d\theta\, d\varphi\, d\nu$$

$$\mathcal{F} = \int_{\nu=0}^{\infty} B(\nu,T) \int_{\Omega} \cos\theta \, d\Omega \, d\nu$$

$$B(\nu,T)=\frac{2h\nu^3}{c^2}\left(\frac{1}{e^{\frac{h\nu}{kT}}-1}\right)$$

Sea $x=\frac{h\nu}{kT}$ $\quad dx=\frac{h}{kT}\,d\nu$

$$\int_0^\infty B(\nu,T)\,d\nu=\int_0^\infty \frac{2h\left(\frac{kT}{h}\right)^3}{c^2}\,\frac{x^3}{e^x-1}\,\frac{kT}{h}\,dx$$

$$=\frac{2h\left(\frac{kT}{h}\right)^4}{c^2}\int_0^\infty \frac{x^3}{e^x-1}\,dx$$

let $z=4$

$$\zeta(z=4)=\frac{1}{(4-1)!}\int_0^\infty \frac{x^3}{e^x-1}\,dx$$

$$\frac{\pi^4}{90}=\frac{1}{6}\int_0^\infty \frac{x^3}{e^x-1}\,dx$$

$$\int_0^\infty B(\nu,T)\,d\nu=\frac{2h}{c^2}\,\frac{k^4}{h^4}\,T^4\,\frac{6\pi^4}{90}$$

$$=\frac{2k^4}{c^2}\,\frac{\pi^4}{15}\,T^4$$

$$\pi\int_0^\infty B(\nu,T)\,d\nu=\frac{2\pi^5k^4}{15c^2h^3}\,T^4=\sigma T^4$$